Stribeck Curves

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Introduction

In the sliding friction of lubricated surfaces, in particular of journal (plain) bearings, the *coefficient of friction* μ cannot be assumed constant as in *Coulombic friction*, but varies with pressure, sliding speed and nature of the lubricant, especially its viscosity. At very low speeds and low viscosity the local microscopic „hills“ called *asperities* touch just as much as if they were unlubricated, but they are „coated“ with a film of lubricant of one or several molecular layers. This is called *boundary lubrication*. The bulk properities of the lubricant don't count, but its molecular structure and weight do. According to Hamrock, Schmid and Jacobson 2004, fatty acids with molecular chains of over 14 units (corresponding to a molecular weight over 120) are particularly good boundary lubricants with μ-values as low as 0.05. This includes myrtistic, palmitic and stearic acids. As speed increases and/or load decreases and/or viscosity becomes higher, μ decreases, eventually by 2-3 orders of magnitude with minima as low as 0.001. Measurements of μ are often expressed as a function of rotational speed (ω, 1/s) times the lubricant's *dynamic viscosity* (η, Pa s), divided by a pressure (p, Pa). The combination ω η / p is dimensionless and called the *Hersey number* H. Several alternative and conflicting definitions of H are in use, as the physical meaning of p is not unambiguous. A journal bearing with diameter d and length l has a sliding area of π d l and a projected area of d l. As the loading force is supported mainly by the bearing surface normal to it, the effective pressure is several times higher than the average pressure calculated with the total sliding area. Using the projected area implies full support over 2 radians (~114.6°), which seems reasonable, although Kidd (2000) in his extensive investigation assumed 90° (~1.57 rad). A plot of μ as a function of H is called a *Stribeck curve*, named after one of the several researchers who discovered in the decades around 1900 the relationships given above. The most prominent feature of a Stribeck curve is the relatively sharp minimum in μ which indicates the onset of complete *hydrodynamic lubrication*, i.e. the point where the wedge of lubricant has enough pressure to completely separate the sliding surfaces so that their asperities no longer touch at all. The transition with strongly decreasing μ-values from the „shoulder“ of boundary lubriction to this minimum is called *mixed lubrication*. After raching the minumum, the μ-values increase gently with increasing speed because of the increasing rate of internal friction movement within the lubricant.

Historical Stribeck-like Curves

The *concept* of the Stribeck curve is well-publicised, but *quantified* curves are rare. A few available ones seem wrong or missing some information. A curve given in Hamrock et al. 2004 shows the characteristic minimum at a Hersey number of about 7, which seems impossibly high using the SI units given. Wäsche and Woydt (2014) avoid a quantified Stribeck curve but show similar families of curves taken from Martens 1889. These vary only one variable at a time: either speed or pressure, with pressure or temperature as parameters. I have tried to construct Stribeck curves from some of this data.

Martens (1889) carried out an extensive evaluation of many lubrication oils, mainly colza or rapeseed oils and mineral oils, and published the „Stribeck-like" curves (before the establishment of this term) shown in Figs. 5 to 7 (original numbering). All the families of curves use more or less the same data, in particular of the refined and de-acidfied oil labeled O_I10, with some of the data shown in Table 5 (original numbering) below. This oil is called „Rüböl", which could mean colza oil or rapeseed oil (canola), but some other oils are called „Rapsöl" (rapeseed oil), so presumably it is colza rather than canola oil. Martens also used a system of averaging and used the term „Rüböl" also for an average of averages of four measured „Rüböl" oils, as a kind of standard for comparison.

Tabelle 5. Haupt-Zusammenstellung über die Versuchs-Ergebnisse. Tabelle 5.

I. Fette Oele.

Erklärung: 1) Zur Erleichterung der Uebersicht sind in Spalte 11 bis 18 die Flüssigkeitsgrade (fe) mit 100 multiplizirt; es fallen also die Decimalstellen fort, fe = 0,97 ist geschrieben 97. Aehnlich so ist der Reibungskoeffizient in Spalte 20 bis 28 mit 100000 multiplicirt angegeben worden, statt $\mu = 641$ ist also zu lesen $\mu = 0{,}00641$. Für das specifische Gewicht sind nur die Decimalstellen angegeben.
2) In den einzelnen Gruppen sind die unmittelbar vergleichbaren mit der neueren Pendelform ausgeführten Versuche vorangestellt. Im Uebrigen ist in Gruppe II und III b die Einordnung nach wachsenden Flüssigkeitsgraden vorgenommen.
3) Die Zahlen in Spalte 37 geben die vermittelst des verbesserten Penskyschen Apparates für den Flammpunkt gefundenen Werthe an; sie sind mit eckigen Klammern versehen, soweit sie mit dem alten Apparat ohne umzurühren bestimmt sind. Die mit runden Klammern versehenen Zahlen in derselben Spalte sind Ergebnisse der Tiegelmethode.

Tabelle 5. a) Pflanzliche Oele. Tabelle 5.

Lfd. Nr.	Zeichen der Probe O Nr.	Bezeichnung der Probe und Art des Oeles	Aeussere Beschaffenheit: Konsistenz	Farbe	Fluorescenz	Durchsicht	Geschmack	Geruch	Specifisches Gewicht	Flüssigkeitsgrad (fe) nach Engler bei den Wärmegraden (Rüböl = 1): 20 C°	30 C°	40 C°	50 C°	60 C°	80 C°	110 C°	150 C°
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
13	I 27	Raff. Rüböl	leichtflüssig	schwach gelb	—	klar auch bei längerem Stehen, setzt keinen Schleim ab	nach raffinirtem Rüböl		914	97	63	44	31	23	15	11	09
14	I 26	dgl.	dünnflüssig	gelb	—	klar auch bei längerem Stehen, setzt keinen Schleim ab	nach raffinirtem Rüböl		914	90	57	40	31	24	16	11	09
15	I 10	dgl. entsäuert	dünnflüssig	gelb	—	klar auch bei längerem Stehen, setzt keinen Schleim ab	schwach nach raff. Rüböl		914	92	62	42	30	23	15	11	09

Lfd. Nr.	Ergebnisse der Reibungs-Versuche: Lagerflächendruck p = / Umfangsgeschwindigkeit v =	10 kg/qmm: 0,5 m/sec.	1,0 m/sec.	2,0 m/sec.	25 kg/qcm: 0,5 m/sec	1,0 m/sec	2,0 m/sec.	40 kg/qcm: 0,5 m/sec.	1,0 m/sec.	2,0 m/sec.	Für v = m/sec.	Pendel: unruhig p =	sehr unruhig p =	Ende des Versuchs p =	Verhältnisszahl für μ (Rüböl = 100): Einzelwerthe	Mittel
1	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
13	Lagerwärme . . . Wärmeüberschuss . Reibungskoeffizient	24,9 1,7 748	24,4 3,4 1373	28,8 6,3 1434	28,3 5,1 635	34,3 13,3 782	35,1 12,6 1075	31,8 8,1 554	39,5 18,3 642	41,4 18,9 802	0,5 1,0 2,0	53 40 —	— — —	66 53 66	— — —	—
14	Lagerwärme . . . Wärmeüberschuss . Reibungskoeffizient	22,4 —1,3 687	22,7 —0,7 1068	27,9 5,4 1389	22,9 —0,6 550	28,1 4,6 733	34,9 12,2 990	23,3 —0,2 458	30,1 6,6 535	37,8 15,1 745	0,5 1,0 2,0	— — 53	— — —	120 80 66	97 91 97	95
15	Lagerwärme . . . Wärmeüberschuss . Reibungskoeffizient	22,2 2,5 732	28,4 4,2 1267	28,4 9,8 1572	22,8 3,2 562	27,5 8,5 825	35,3 16,5 1100	23,8 4,1 447	30,9 11,7 619	38,4 19,6 829	0,5 1,0 2,0	— 80 —	— — —	120 93 66	97 104 106	102

Table 5 (original numbering): Excerpts from Martens 1889

Figure 5 (a modified excerpt of the original Fig. 5) plots this data as three curves for sliding speeds of 0.5, 1.0 and 2.0 m/s. The x-axis represents pressure increasing in steps of 20 kgf/cm² from 0, the y-axis represents the coefficient of friction μ increasing in steps of 0.002 from 0. Table 5 only gives data up to 40 kgf/m² ; the rest is given in Table 6 on page 81 of the reference, which also shows the temperatures of the bearings. These consist of three parts made of red brass and the top part can be loaded with pressure.

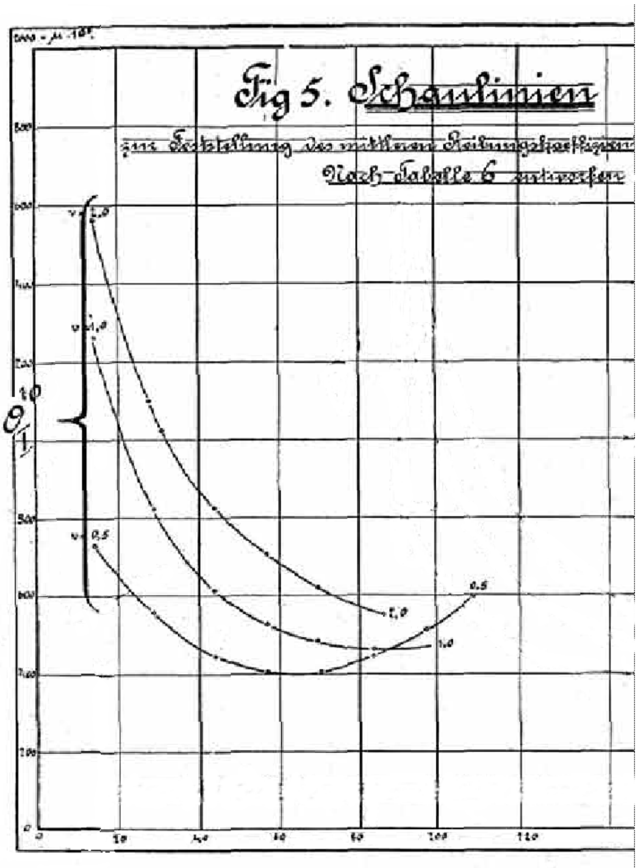


Figure 5 (original numbering): Excerpt from Martens 1889.

Figure 7 is similar, but shows 9 curves for temperatures from 20°C to 100°C, all at 0.5 m/s speed or 1.66 rps (see explanation in the next section).

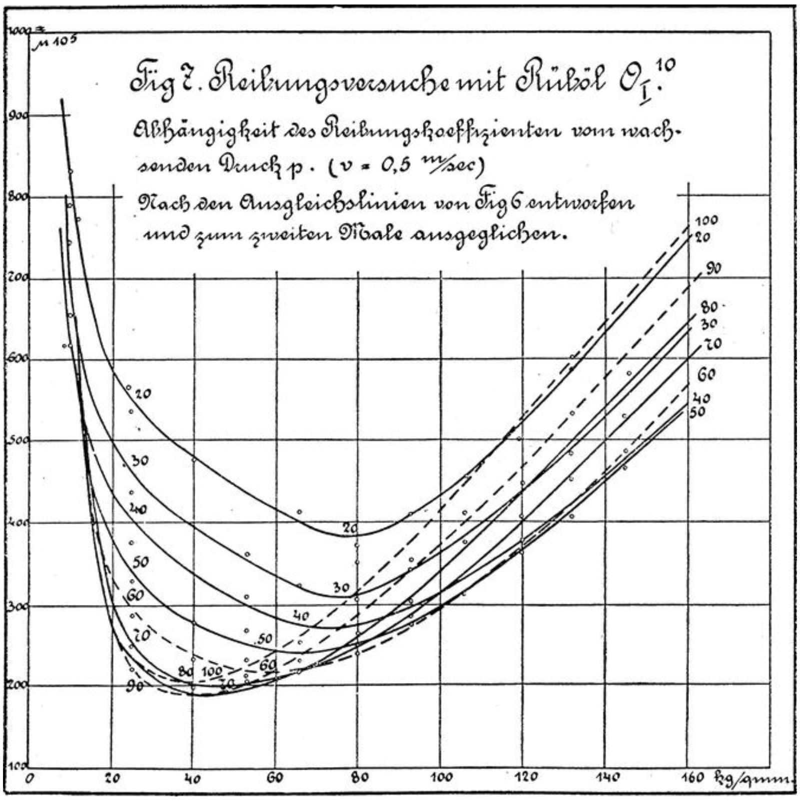


Figure 7 (original numbering): μ vs. p. (From Martens 1889)

Both figures 5 and 7 represent the same oil ($O_I 10$) but only fig. 7 holds the temperature constant for each curve and clearly uses more data than given in Tables 5 and 6. Indeed the caption of Marten's fig. 7 says the data comes from the curves of *figure* 6. This shows 11 sets of data points, plotted as a function of rising temperature. Each set represents a pressure and is overlaid with a smoothed curve.

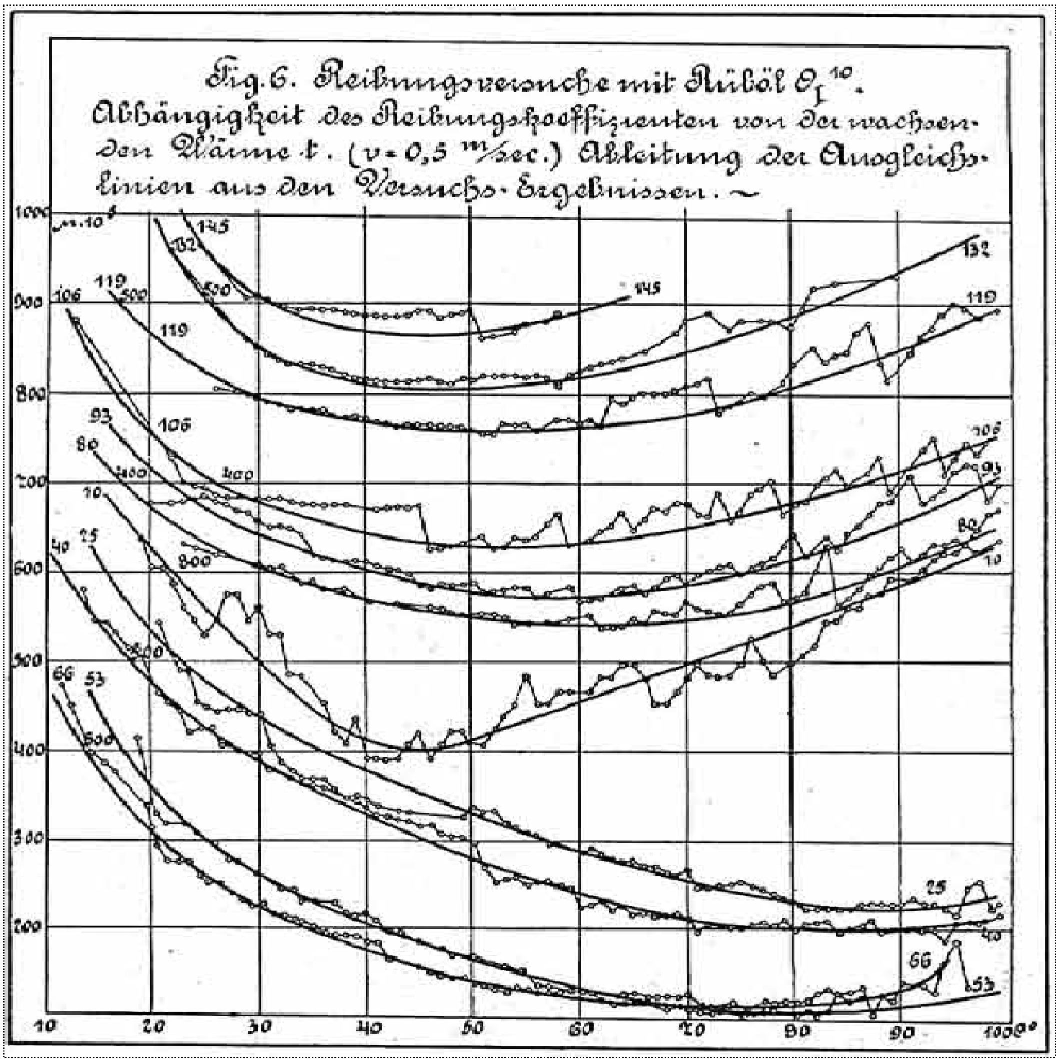


Figure 6 (original numbering): μ vs. T. (From Martens 1889)

A further diagram in the reference (not shown here) plots curves of constant μ with pressure on the x-axis and temperature on the y-axis.

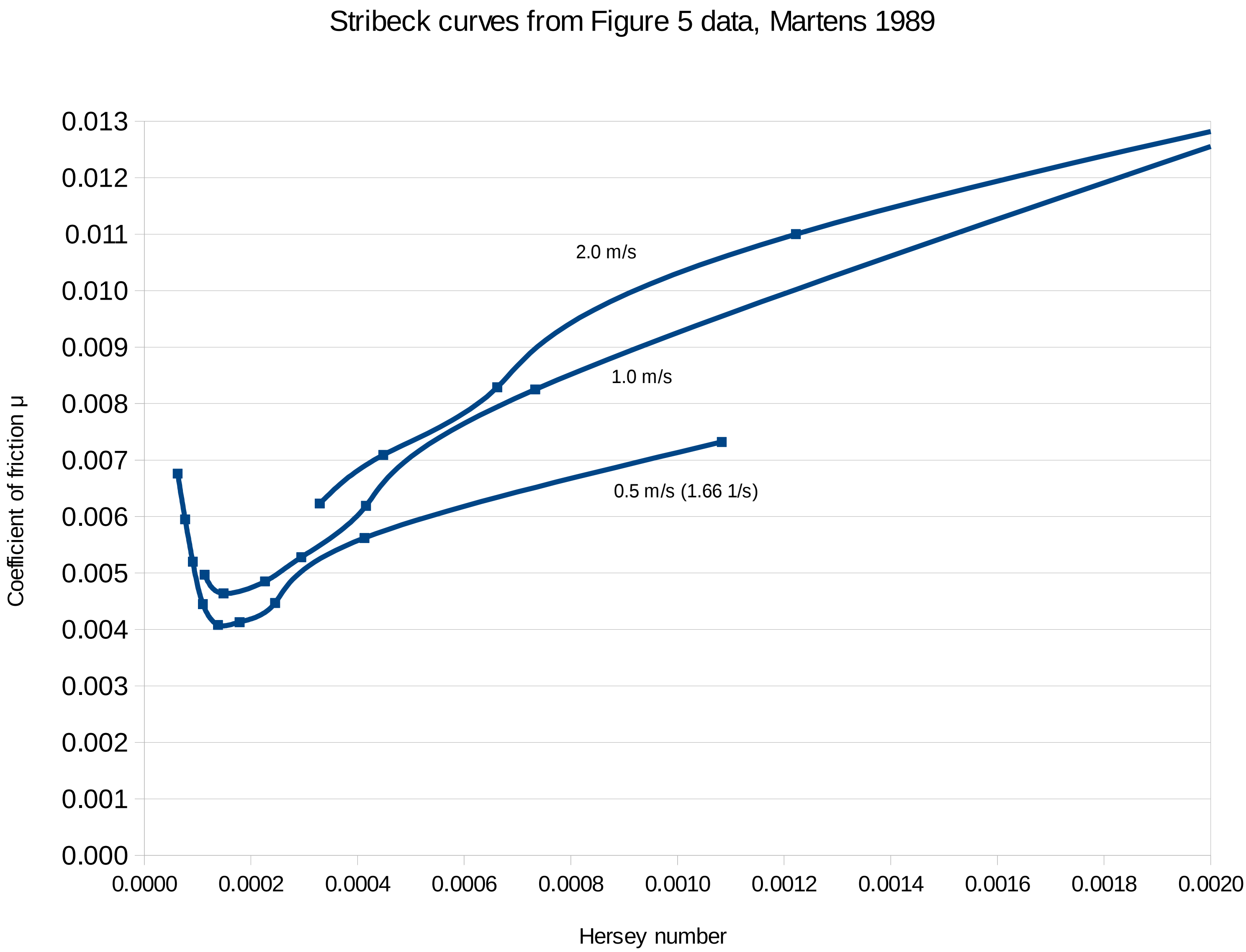
Stribeck Curves from the Martens Data

In order to get actual Stribeck curves, i.e. plots of μ as a function of the Hersey number, from the Martens 1889 data, we need to have the (dynamic) viscosity values as a function of temperature. Table 5 gives some values, but in units of degrees Engler (°E). Although it is possible to convert °E into the SI unit for *kinematic viscosity* (m^2/s) e.g. using https://www.ksb.com/centrifugal-pump-lexicon/kinematic-viscosity/191106/ (and then convert into dynamic viscosity η by multiplying with the the density), this is inaccurate at low viscosity values and no conversion at all is given for less than 1°E. The values given by Martens are all below 1°E, so it is unclear how to convert them. Martens (1888) does explain exactly how he measured viscosity, but rather than spend hours studying this text, I chose to use modern viscosity data. The values for vegetable oils are all rather similar and are likely to be nearly the same as those of the oils Martens used. Sahasrabudhe et al. (2017) give values of η in units of Pa s for temperatures from 20°C to 200°C (see fig. 3 there). They also give a power curve fit for this: η (Pa s) = 8.3 T(°C)^-1.52 (for canola oil), but a more accurate fit is η = 0.116 – 0.00336 T + 0.00004336 T^2 – 0.0000002922 T^3 + 0.000000001 T^4 – 0.00000000000137 T^5 or simpler η = 0.085 / (1 + (0.03 T)^2.2) + 0.001. (I'm always amazed how curves can often be fitted quite accurately with quite different mathematical functions; the last one is a simplified 4-parameter sigmoidal.)

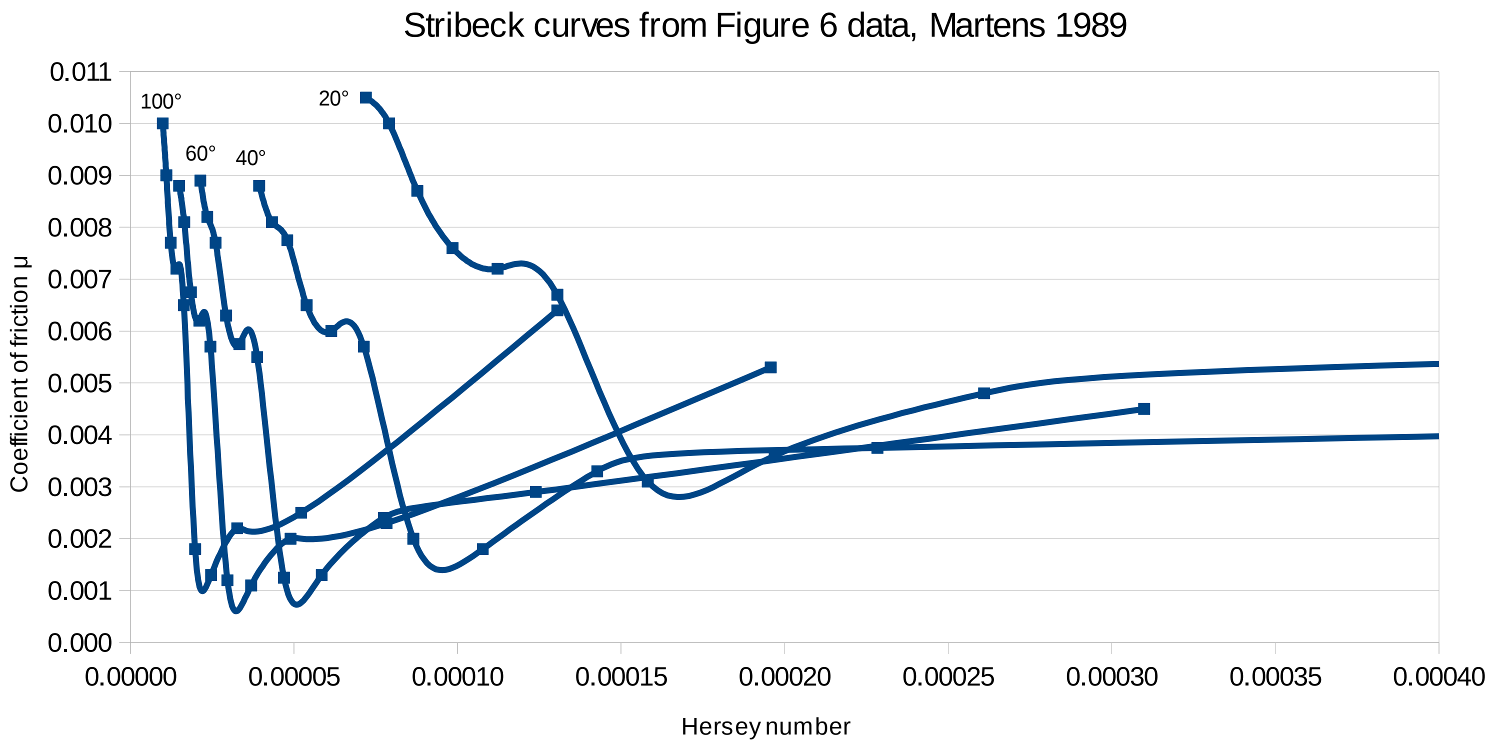
We also need the pressure relative to the bearing's projected area. The bearing dimensions of Marten's testing machine are l = 0.07 m and d = 0.0996 m, giving a projected area of ~0.007 m² and a sliding area of ~0.022 m². The pressure values that Martens supplies in kgf/cm² seem to relate to the top part of his three-part bearing, and if this covers 120°, it would relate to one third of the sliding area, i.e. ~0.0073 m². This is close enough to use his values directly, only converting them to Pascals (Pa).

For the speed, full data in figures 6-7 are available for one sliding speed, 0.5 m/s, but figure 5 and table 5 also supply some data for 1.0 and 2.0 m/s. With the shaft diameter of 0.0996 m, this gives rotational speeds of ω = 1.66, 3.33 and 6.66 1/s.

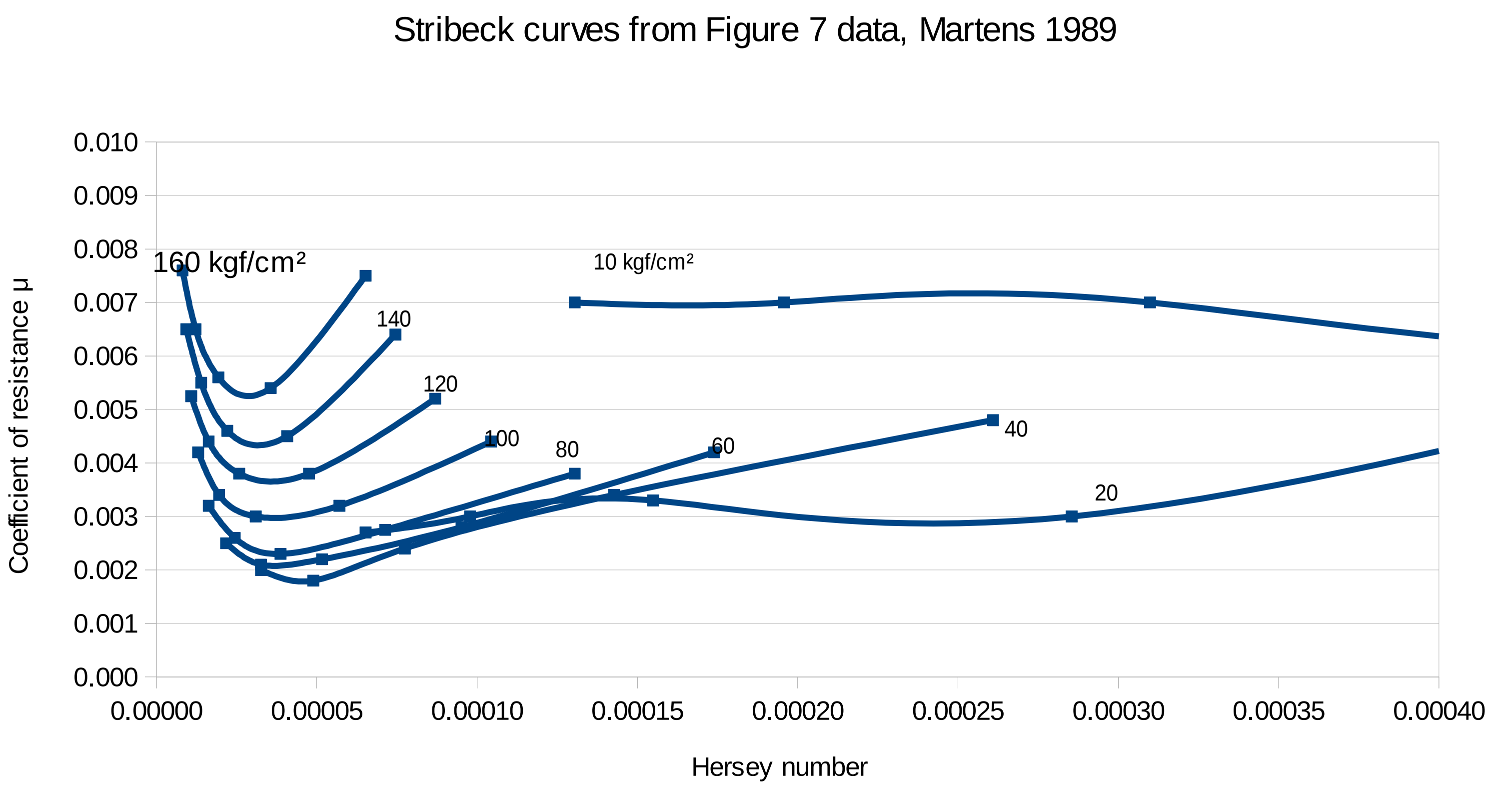
Therefore we now have all the parts of the Hersey number and can plot actual Stribeck curves. Wishfully all of Marten's measurements of a single oil (his O_I^{10}) would result in a single Stribeck curve. Actually this isn't the case: we get a set of curves, and even different sets depending which of his figures is „converted“. The first figure below shows Marten's figure 5 converted to Stribeck curves. They show a nice minimum of μ around 0.004-0.005 at a Hersey number of about 0.00015. The greatest variation in the underlieing variables is in the pressure, which varies in a range of about 10:1. The speed varies in a range of 4:1, and the viscosity at most 2:1.



The next figure below shows Marten's figure 6 converted to Stribeck curves. Although the characteristic shape is shown, both the values of minimum μ and the Hersey numbers at the minima are about half of the corresponding values in figure 5, and higher temperatures shift the curves strongly towards smaller Hersey numbers. All curves are for a speed of 0.5 m/s. The greatest variation is in the pressure, which varies in a range of about 14:1. The viscosity varies in a range of 8:1.



The next figure below shows Marten's figure 7 converted to Stribeck curves. Although the data is the very same as in figure 6, the different presentation gives quite differently shaped curves with a large varation of the minimum μ at Hersey numbers from about 0.00003-0.00005. All curves are for a speed of 0.5 m/s.



Conclusion

The wishful goal using a Stribeck curve to simplify the graphical data of oil/bearing measurements was not met. The Hersey numbers seem exceedingly low and the variation in the curves high. Perhaps I made some serious mistakes. But maybe the data and figures are correct and the Hersey number is simply not very useful for this purpose. Interested readers are encouraged to examine the data available in this spreadsheet. Please report errors or new insights.

References

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