Efficiency of human-powered sail pumping

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ABSTRACT

Sail pumping involves oscillating a sail repeatedly in a rolling and yawing motion about the mast base. The motion is similar to that of aquatic animals such as fish and turtles propelling themselves by oscillating their fins or flippers.

We find that humans performing sail pumping at cruising speeds in zero-wind conditions naturally stroke at a Strouhal number that is in the same range used by turtles, fish and birds, a range known to produce efficient propulsion. We postulate this occurs from subtle feedback to the pilot consisting of excessive effort for stroking outside the correct range.

Stroking in the correct Strouhal number range does not guarantee the highest possible efficiency, only the best efficiency using a given propulsion design. Indeed, the efficiency of pumping with windsurfer sails was found to be about 20%, which is considerably lower than laboratory results for rigid oscillating and flapping foils. We believe the cause of the inefficiency may be the poor aerodynamics of the windsurfer sail, the limits to stroke amplitude imposed by human arm length, and angles of attack that were too large during testing.

We also find that the heart rate can be an accurate method for determining human power output. Sail pumping efficiencies calculated using the heart rate were within 2-3 percentage points of efficiencies measured with a PC-based force data acquisition system.

INTRODUCTION

While windsurfers routinely oscillate their sails (a process known as “sail pumping”) to increase speed in light winds or to provide a burst of acceleration to hasten the transition to planing, sail pumping has been the topic of research only in the context of the physiological response to the pumping. No research that we are aware of has been conducted on the efficiency of pumping sails.

As environmental concerns become more important, sail or airfoil pumping may become a viable method of propelling future sailing ships when wind conditions are light, due to very high efficiencies in light wind conditions. Once wind conditions are stronger, the foils could be used as sails to provide propulsion.

In this work we investigate the efficiency of human-powered sail pumping in zero wind conditions using a windsurfer sail and mast/boom assembly mounted on a wheeled cart, as shown in Figure 1. We note that the sail is not completely rigid, and twists as the sail is pumped. The right photo in Figure 1 shows the aft upper portion of the sail twisting as the pilot pushes the sail away from him. The left photo is taken in-between strokes.

The bottom of the mast is attached to the platform using a windsurfer articulated joint, and the mast rolls from side to side to provide translational motion to the sail as the sail simultaneously yaws about the mast. A video of the sail pumping motion may be seen at https://www.youtube.com/watch?v=2fZXNxqJbtQ and https://www.youtube.com/watch?v=VpxNAKokcMg . The general term "wing" will be used to denote what is being oscillated: a sail, fin, airfoil (foil) or flipper.

Figure 1. Windsurfer cart under sail pumping power. In the photo at right one can see the sail twists at its aft upper end due to the stroking motion away from the pilot.

There are two general types of stroking methods: oscillation and flapping. In oscillating motion, the leading edge of the wing is attached to the body of the craft, as in the tail fins of fish, whales and dolphins. Figure 2 shows flapping motion by a sea turtle, in which the wing is attached to the body at the wing's root or base. Sail pumping involves flapping since the mast is attached to the base of the sail.

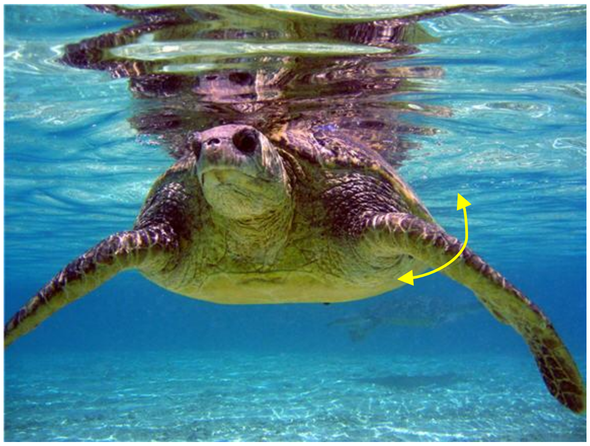

Figure 2. Sea turtle uses a flapping motion, with motion about root of wing. Photo by Mark Sullivan, NOAA, www.nmfs.noaa.gov/pr/species/turtles/photos.html.

The pitch θ is the angle of the wing relative to the direction of motion, while α is the angle of the wing into the resultant of the translational motion of the wing and the forward motion of the vehicle, as shown in Figure 3. The angle α is referred to as the angle of attack of the wing. From an aerodynamic point of view α is obviously more important.

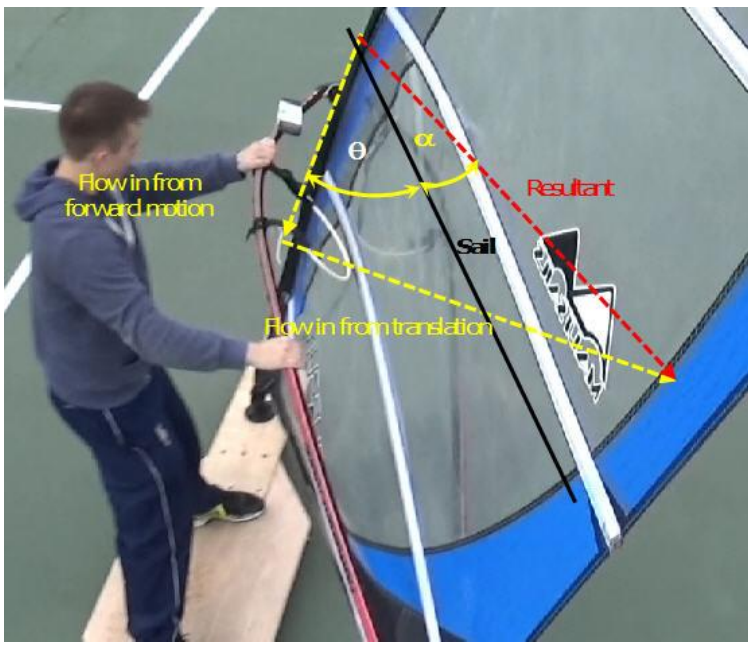

Figure 3. Sail angle of attack and pitch.

The translational displacement of the wing is referred to as heave h, and has a value of zero at the midpoint of the foil's translation. The maximum heave is denoted by h_o and represents one half of the total amplitude of the foil translation. The heave is normally measured at the pivoting axis of the wing yawing point. The parameter h_o/c is used in this and other studies as a dimensionless parameter for the heave.

The phase angle φ is an important parameter, and describes the difference in phase between the pitch of the wing θ as it leads the heave of the foil h. A phase angle of 0 degrees results in the heave and pitch angle reaching their maximum values at the same time (the end of the stroke), while a 90 degree phase angle results in the wing at its minimum angle when the heave reaches its maximum at the end of a stroke.

Another very important parameter is the Strouhal number St, given by:

$$St = f\ A/V \tag{1}$$

Where f is the frequency of oscillation of the wing, A is the amplitude of the stroke, usually measured at the point of the foil rotation, and V is the speed of the craft. The Strouhal number can also be thought of as half of the ratio of the average translational speed of the wing to the speed of the craft.

Since the mast and sail in this work are rolling in an arc about the bottom of the mast, the sail oscillates with an amplitude that depends on the height over the bottom. The sail then experiences a range of Strouhal numbers. For the purposes of this study we will report the Strouhal number using the amplitude at 0.7 times the sail height or span, as this was the method used in the flapping foil literature to capture properties at the center of effort of the sail. The amplitude is measured at the leading edge of the sail (the center of rotation).

In addition, one may consider a Strouhal number St_{TE}

$$St_{TE} = f\ A_{TE}/V, \tag{2}$$

which uses the maximum translational amplitude A of the foil occurring at the trailing edge of the foil. Studies of Strouhal numbers made on aquatic animals and birds typically use this version of St. We will calculate this parameter using the maximum amplitude, which occurs at the trailing edge of the sail.

A thrust coefficient Ct is defined as:

$$Ct = 2 * Thrust/(\rho V^2 A) \tag{3}$$

Where thrust is the force in the direction of vehicle motion, rho is the air density, V is the velocity of the vehicle, and A is the sail area.

PREVIOUS WORK

Read (2001), Read et al. (2003), and Hover et al., (2004) have found that computer-controlled rigid oscillating foils can achieve high propulsive efficiencies (we stress that oscillating foils and flapping foils are different). The experiments of Anderson et al. (1998) show significantly higher efficiencies than Read and Hover, as high as 0.86 at St = 0.3, as compared to 0.72 at St = 0.16 for Read and 0.64 for Hover at St= 0.25. All the researchers found that the Strouhal number, maximum foil angle of attack and phase angle are important parameters for predicting efficiency.

The researchers also discovered that the maximum heave to chord (h_o/c) ratios they tested (1 for Read and Hover, and 0.75 for Anderson) resulted in peak efficiencies, as did maximum angles of attack α between 15 and 20 degrees. In addition, they found that a peak in the efficiency is found when the phase angle is around 90 degrees. Experiments performed by these researchers were performed with the foil heaving linearly, with end caps at the foil tips to simulate an infinite aspect ratio.

Read concluded that even very small changes in the timing of the angle of attack can have a significant impact on efficiency and thrust production. Hover experimented with a variety of stroke-timing schemes, concluding that controlling the foil angle of attack with a sine function (as opposed to controlling the pitch angle with a sine function) gives optimal results. In his thesis, Read (2001) published values of forces perpendicular to the direction of the thrust forces, and found these forces could range from about 1 to 4 times greater than the generated thrust forces.

Liu and Bose (1997) found that adding span-wise flexibility to an oscillating foil could substantially increase efficiency if the flexibility is actively controlled during stroking. They found that efficiency could be increased from 78% to 83% over rigid foils. Prempraneerach et al. (2003) showed that adding chord-wise flexibility could increase efficiency of rigid foils by 36%. Riggs et al. (2010) found that a foil with varying chord-wise stiffness resulted in an increase in efficiency of 26%.

For foils undergoing flapping motion (more similar to the windsurfer rig), Polidoro (2003) found that a foil aspect ratio of about 4 provided optimal thrust. McLetchie's (2004) experiments showed flapping foil efficiencies that were within the range found with oscillating, rigid foils that use end caps: 80% with ho/c ratio of 1, a maximum angle of attack around 15 degrees, and a Strouhal number of 0.3, measured at 0.7 times the span of the foil.

Licht et al. (2010) discovered that flapping motion that includes additional in-line movement of the foil along the direction of craft motion can result in significant increase in efficiency over normal symmetrical flapping at maximum foil angle of attack of 40 degrees. The motion involves a thrust producing stroke downstream and a feathered stroke upstream, and mimics the stroking of sea turtles.

Izraelewitz et al. (2014) demonstrated that in-line motion could be adjusted to eliminate or greatly reduce the very strong lift forces perpendicular to the thrust that appear with symmetrical flapping. In addition, the authors were able to generate pure lift without any thrust, as well as combinations of the two forces.

Nature also appears to pay close attention to the Strouhal number. Triantafyllou et al. (1991), Triantafyllou (1993) and Rohr et al. (1998), reported that dolphins, sharks, and bony fish swim at $0.2 < St_{TE} < 0.4$, and Taylor et al. (2003) showed that birds, bats and insects cruise within this same range as well.

The high efficiency results of Read (2001, 2003), Hover (2004), and Anderson (1998) are accompanied by low thrust coefficients. However, by increasing the Strouhal number the thrust coefficient can be made to increase.

METHODS

Experiments were conducted using a Gaastra 6.5 m^2 windsurfing sail mounted on a large skateboard using a conventional windsurfer mast and boom assembly, or rig. The resulting oscillations classify as flapping, although due to the twisting of the aft portion of the sail (as seen in Figure 1), there can be some profound differences between sail and rigid foil flapping, as discussed later in this text. The sail height is 4.5 m, and the chord length at its widest point is 1.7 m, with an average chord of 1.1 m. The average aspect ratio of the sail is 4.1. The center of effort (CE) of the sail is located 2 m over the deck.

The center of effort is taken to be the same as the centroid height of the sail, which is 44% of the sail span. This value is consistent with McLetchie's (2004) high angle of attack data.

Experiments were first conducted on a windsurfer, but we found that the oscillations caused by the side lift forces caused considerable periodic yawing which adversely affected efficiency. Rather than modifying the windsurfer board in order to minimize this effect, we decided to conduct our experiments on land, using a large skateboard constructed for this purpose.

We determined the total drag force (rolling resistance plus air resistance) on the board and rider by measuring the tension on a spring scale pulled by a bicycle at various speeds over a wooden gymnasium floor. The tests were performed with the rider on the board, without the sail rig. The sail rig was omitted because we experienced difficulties in maintaining the sail in a neutral position. We were able to confirm the accuracy of our measurements by running a second set of experiments in which we analyzed time and position plots of the rig, obtaining drag values from Newton's 2nd law.

The drag force from the sail rig was calculated by treating the rig as a vertical cylinder with a drag coefficient (Cd) of 1, using the average diameter of the mast. This is an appropriate Cd at the Reynolds numbers experienced during testing (White (2015)). We then added the rig drag to the drag measured without the sail. Figure 6 shows the drag force of the board and rider with and without the sail.

Velocity values were obtained by measuring the time elapsed to cover a distance of 12 m on a gymnasium floor, after a 12 m acceleration run to ensure constant speed during the timed portion.

Figure 4 shows the total drag force results using a 6 ft, 170 lb rider with and without the sail rig, as a function of speed. The data for the total drag are well-fit by a 2nd degree polynomial: $Drag(N) = 1.79934 - 0.0918752\, V\ \left(\frac{m}{s}\right) + 1.91258\, V^2$.

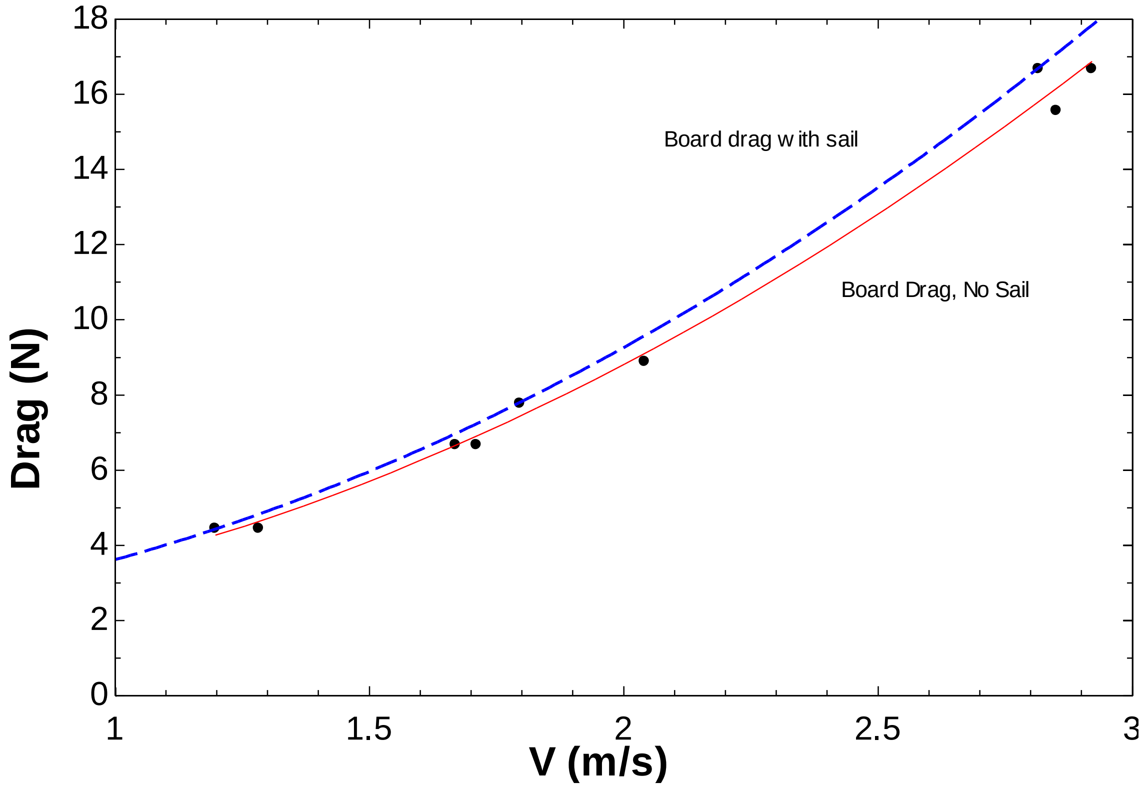

Figure 4. Drag force on rolling board with and without sail rig.

It appears that the experimental drag data are consistent to within about +- 0.75 N at higher speeds and about 0.25 N at lower speeds. Since test speeds ranged between 2 and 2.7 m/s the error in measuring the drag force was probably around 5%.

We measured force applied to the windsurfer boom by a pressure sensor connected to an inflated bladder (Figure 5). As force was applied to the bladder, a laptop computer carried on the pilot's back recorded pressure readings every 0.003 seconds. The sensor was calibrated by hanging weights from the bladder. The sensitivity of the instrument depended on the inflation pressure, and a pressure of 18.5 psig was found to be high enough to prevent bottoming out of the bladder when pressure was applied while still providing the required sensitivity.

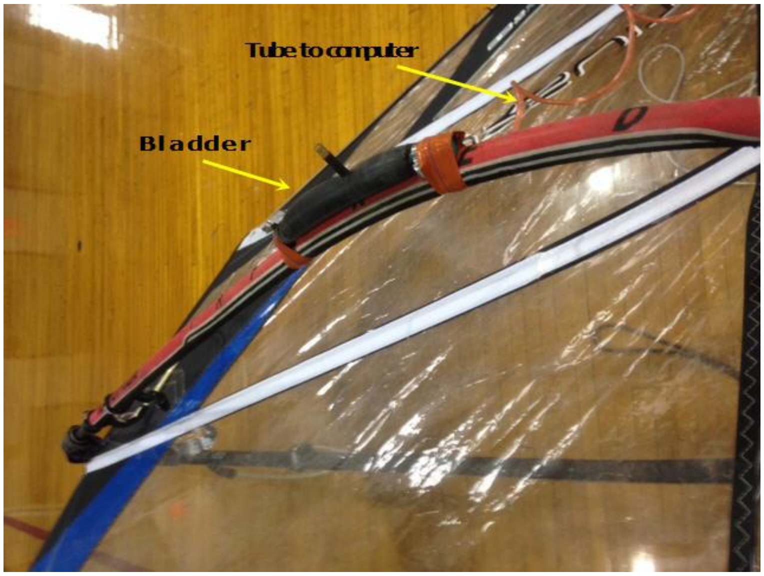

Figure 5. Bladder for measuring force applied by pilot. Bladder is attached to the boom. In this configuration the pilot would push on the bladder as he strokes. For measuring the pulling force the bladder would be rotated around the boom 180 degrees.

Only one sensor was used. The sensor was secured to the boom at a position where the pilot's hands would grip, followed by 2-4 runs up and down the gym floor. At the end of each run, the sensor location was moved until all four sets of data were taken (left and right arms in push and pull configurations). The power input to the boom was calculated by force multiplied by the stroke velocity.

We also measured pilot power output by measuring heart rates. Achten and Jeukendrup (2003) report that the heart rate can be a reliable method of determining human power output, but care must be taken to avoid "cardiac drift", where the heart rate increases after 5-10 minutes of uninterrupted exertion. Hilliskorpi et al. (1999) showed that the mode of the exertion was not important as far as predicting power output – heart rate measurements from hand cranking, pedaling or rowing should give similar power results. We confirmed this by comparing the power output from a cycling dynamometer with that from an elliptical trainer using hand and leg motion, resulting in virtually identical power results from the same heart rates.

Heart rates were measured using a Polar heart rate monitor with a chest strap. We measured the relationship between power output and heart rate for the pilots using 2 Matrix exercise bicycles. Figure 6 shows the relationship between power and heart rate for two pilots used for the study. A total of three pilots were used.

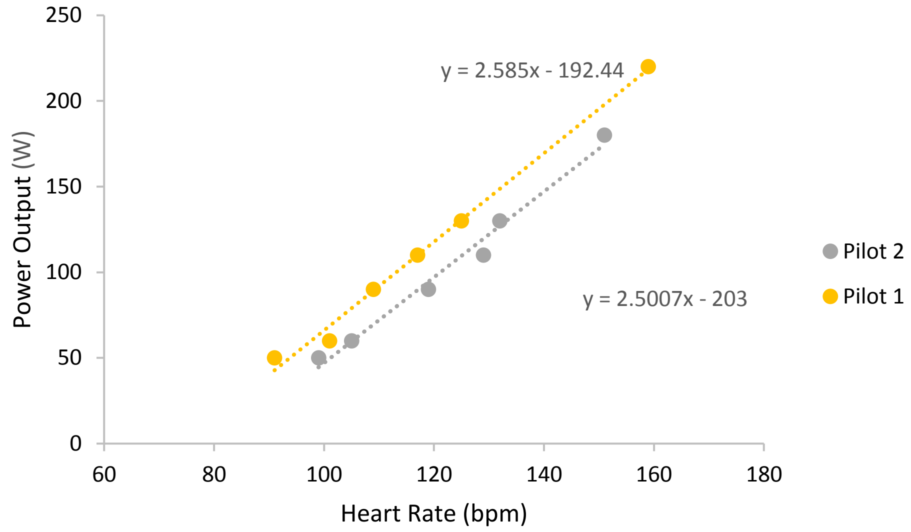

Figure 6. Heart rates of two pilots.

The accuracy of the exercise bicycle power measurements is specified by the manufacturer as within 5% of laboratory dynamometer readings (Matrix (2016)). According to the manufacturer, all of the products are tested before they are shipped.

The efficiency of the stroking was calculated by:

$$\eta = \frac{power\ output}{power\ input} = \frac{Thrust\ x\ Velocity}{Power\ Input\ from\ heart\ rate} \tag{4}$$

The thrust developed was determined from Figure 4, with thrust assumed equal to drag.

The windsurfer rig rotates about two axes while oscillating back and forth, as shown in Figure 7.

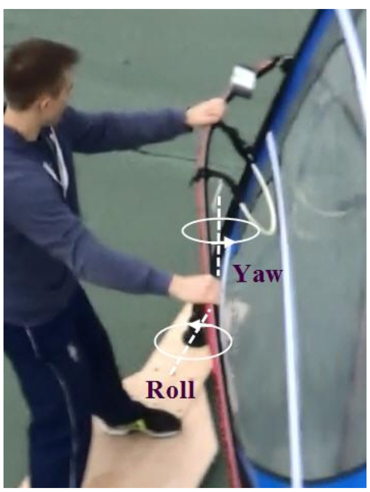

Figure 7. Sail rotation axes.

We used the 3-D gyroscopes on an iPhone 5s to measure yaw and roll of the rig, with a SensorLog app for processing and displaying the data. The sensor output was tested against measurements made with slow motion videos of sail pumping, as well as experiments in which the yaw and roll were simultaneously changed through arcs of the same magnitude encountered in our runs. The sensor values were found to be well within 5 degrees of measured angles. Data were collected at a sampling rate of 30 Hz, and uploaded to a PC after a run was complete.

The yaw data from the sensors were used directly to calculate the pitch angle theta. The heave (translational position of the sail) was calculated by simple geometry relating the roll angle to displacement. Translational velocity of the sail was calculated by computing the change in heave readings between samples divided by the sampling time. The angle of attack was calculated at each sampling time by finding the resultant of the incoming air speed into the board and the translational velocity of the foil.

None of the pilots was aware of his Strouhal number at the time the experiments were conducted. The pilots were not world-class athletes, but individuals in fairly good physical condition. Pilots 1 and 3 were about 20 years of age, while the age of Pilot 2 was 3 times that of the younger pilots.

RESULTS

All the pilots experienced some initial difficulties learning to stroke properly, and relied on a process of trial and error to achieve an acceptable level of fluidity and control. This took about one half to one hour of cumulative practice over several sessions. Moving the sail in a stroking motion was a highly aerobic undertaking at the beginning, with resulting heart rates over 150 beats per minute. After some experience, the pilots were able to stroke in a more relaxed mode, with typical sail pumping heart rates between 106 –120 beats per minute.

The results for two pilots are summarized below, and show that the efficiencies measured using the heart rate are within 15% of those measured using the sensor. This is an important finding for further tests that may be conducted on the water, because of the ease with which heart rate data can be taken.

Pilot	Efficiency (heart rate)	Efficiency (sensor)	St (TE)	St (0.7)	Phase Angle (degrees)	$\frac{h_o}{c}$ (0.7)	$\frac{h_o}{c}$ (0.44)
1	15.6 %	18.2%	0.5	0.4	100	0.3	0.2
2	15.8%	17.8%	0.4	0.3	70	0.3	0.2

Table 1. Results of experiments. TE refers to the trailing edge, 0.7 is at 0.7 times the sail height, 0.44 is at the center of effort of the sail, or 0.44 times the mast height.

The table shows that the pilots stroked in about the range of St_{TE} as found in nature (0.2< St_{TE}<0.4 (Taylor (2003)), a range shown to be the most efficient, and in the same range of phase angle found efficient in Anderson, Read, and Hover.

We now address the efficiency results, which are quite low compared to the maximum efficiencies cited in the literature for oscillating and flapping foils. Using a hot-wire anemometer, we measured the maximum speed of the air exiting the sail as 4 m/s relative to the board, with the board moving at 2.25 m/s relative to the ground, at about 1-2 m behind the sail and 2 m above the deck. From this we calculate a Froude efficiency:

$$\eta = \frac{1}{(1 + 0.5\left(\frac{V_{increase\ thru\ prop}}{V_{board}}\right))} = 0.72$$

The Froude efficiency assumes flow is linear and frictionless, and considers only the losses due to the discharged stream's kinetic energy behind the propulsor. The equation shows that a propulsor is most efficient when it increases the speed of the incoming fluid as little as possible.

According to Stinton(1984), modern aircraft propellers can achieve efficiencies that are about 0.8 times the Froude efficiency, which suggests that sail pumping should have an efficiency of about 58%. It is clear that at 20% efficiency, there are irreversibilties in sail pumping that cannot be attributed to excess exit velocities.

It appears that the phase angle, exit velocity and Strouhal number can be eliminated as possible causes for the seemingly low efficiencies, and we next address the maximum angle of attack.

The average maximum angle of attack at the 0.7 span height (4.5 m) was calculated as 23 degrees. The angle of attack had to be corrected for the twist of the sail, which greatly reduces the angle of attack. Without the twisting of the sail, the angle of attack would have been about 60 degrees.

A direct comparison with McLetchie's data is then not entirely appropriate, as our wing was twisting, while McLetchie's was rigid. Because of this, McLetchie's wing has increasing angle of attack as height increases, while our sail has a decreasing angle of attack. The result is a lower center of effort for our sail, which we estimated at about 0.44 of the mast height (2 m). Figure 8 shows the angle of attack of our sail at a 2m height, which should give a more realistic measure of the windsurfer sail performance. The average maximum angle of attack for the sail pumping is about 50 degrees.

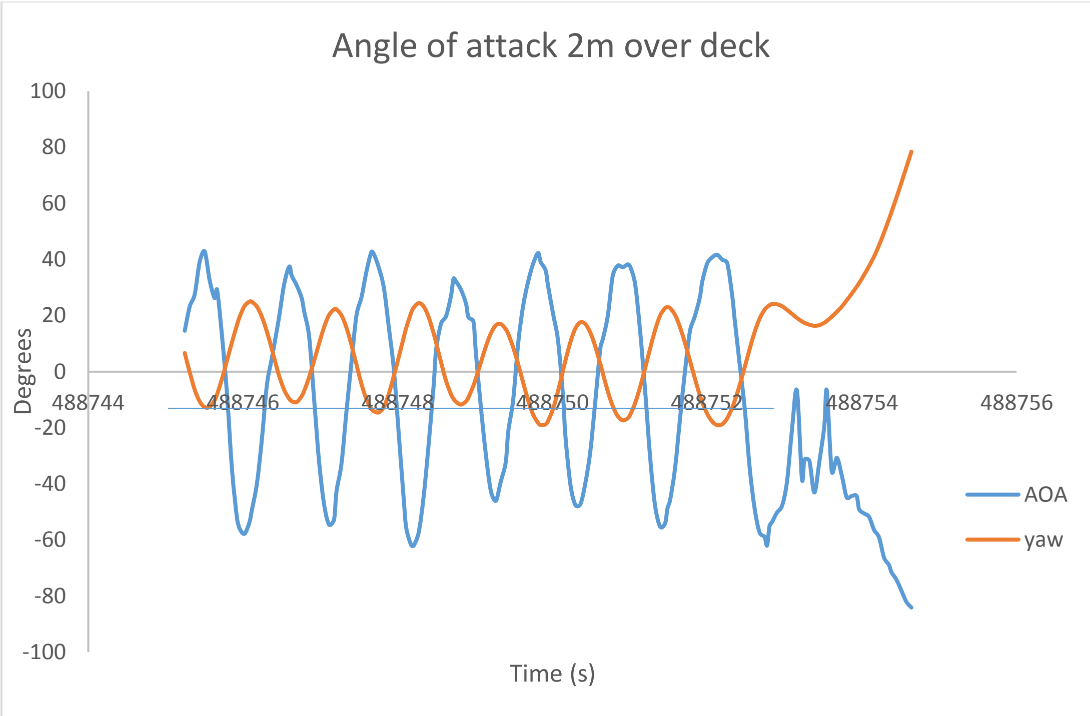

Figure 8. Angle of attack 2 m over deck (Center of Effort).

Figure 9 shows the efficiency of flapping foils by McLetchie, using a Strouhal number of 0.3. Our efficiency results are shown by the green triangle on the plot, at a ratio h_o/c of 0.4. It is clear that our results are reasonable if McLetchie's and data are extrapolated to 50 degrees angle of attack and h_o/c of 0.4.

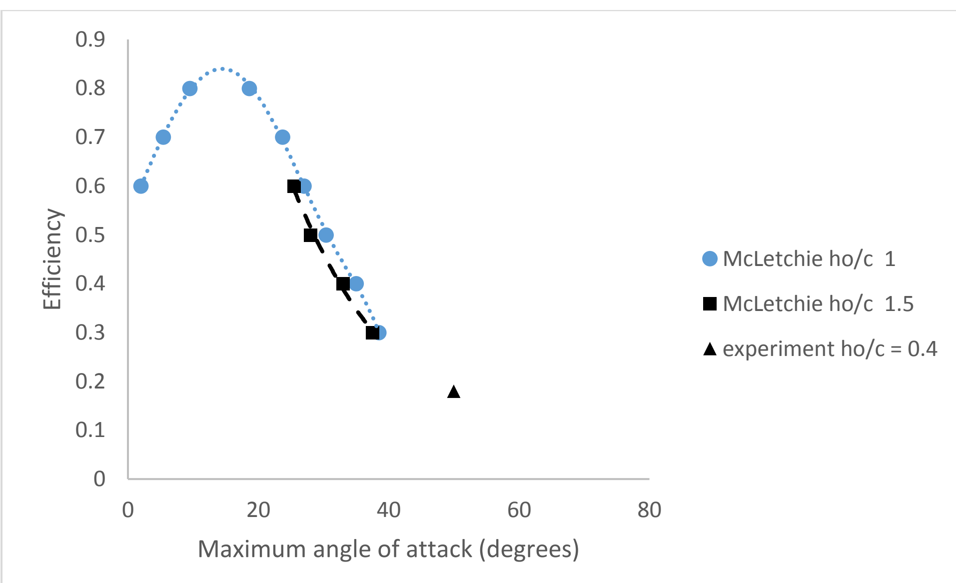

Figure 9. McLetchie's flapping foil data show that efficiency is compromised at higher maximum angles of attack. Our results are shown by the triangle.

Figure 10 shows the results from various experimental runs in which the stroking style was varied. The results include very slow stroking, stroking with a feathering return, stroking with more exaggerated pitch, and normal stroking. A peak in the efficiency can be seen at Strouhal number of 0.3 for the normal stroking of Pilot 2. This peak is similar to that found in the literature results at about the same Strohual number.

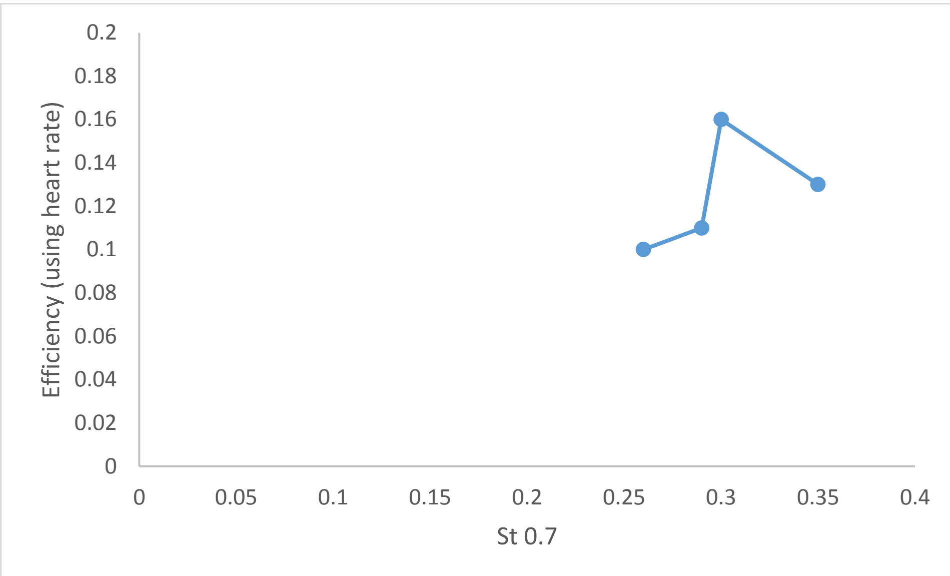

Figure 10. Peak in the efficiency at St = 0.3.

McLetchie's results were performed at h_o/c (0.7) ratios of 1, while our results used only 0.3 due to the limits of the arm length of the pilots. Figure 11 below shows the effects of h_o/c on efficiency, from Anderson's oscillating foil experiments. It appears from this plot that a reduction in h_o/c (0.7) from about 1 to to 0.4 would result in an efficiency loss of about 13%. But, if we consider that the h_o/c (0.44) of 0.2 may be more important because it better represents the center of effort, then the reduction in efficiency from h_o/c of 1 to 0.2 is about 20%.

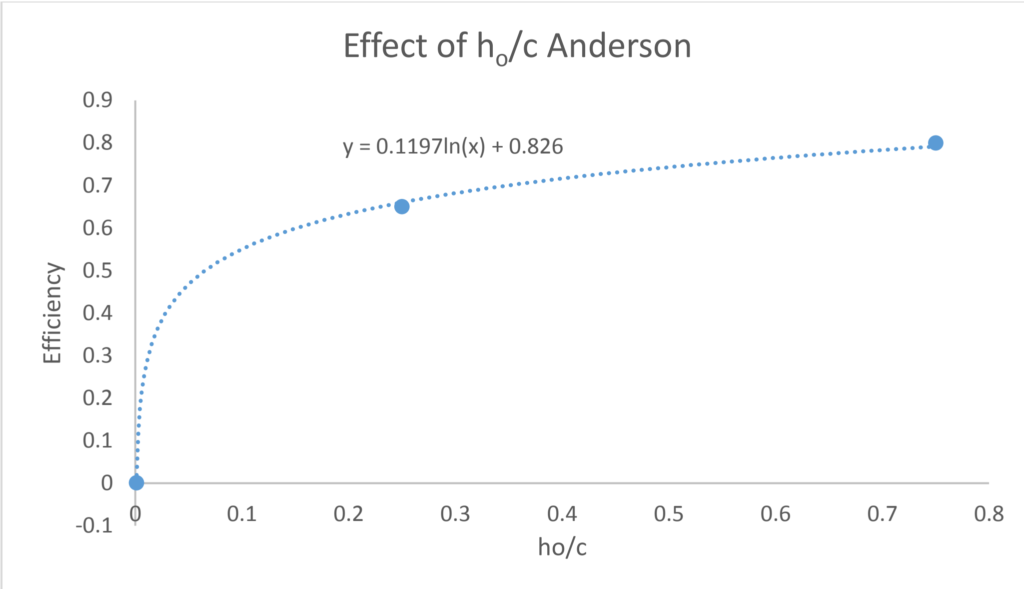

Figure 11. Effect of h_o/c on efficiency, per Anderson for oscillating rigid foils.

Figure 12 shows McLetchie's thrust coefficient (Ct) results as a function of angle of attack at St of 0.35 and 0.25. Our results show thrust coefficient is in the correct range at the maximum angle of attack of 23 degrees at 0.7 times the mast height. However, at the more important center of effort with a maximum angle of attack of 50 degrees, our Ct values seem somewhat high.

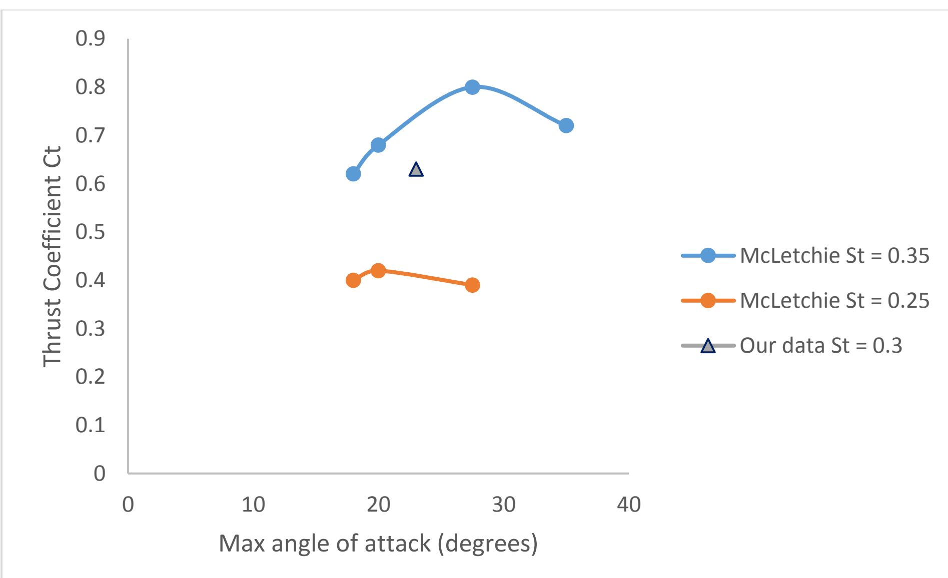

Figure 12. Thrust coefficients for oscillating foils for our results and McLetchie.

In Figure 13 we show the wattage expended by Pilot 2 as a function of stroking speed and velocity of the cart. Other pilots had very similar results.

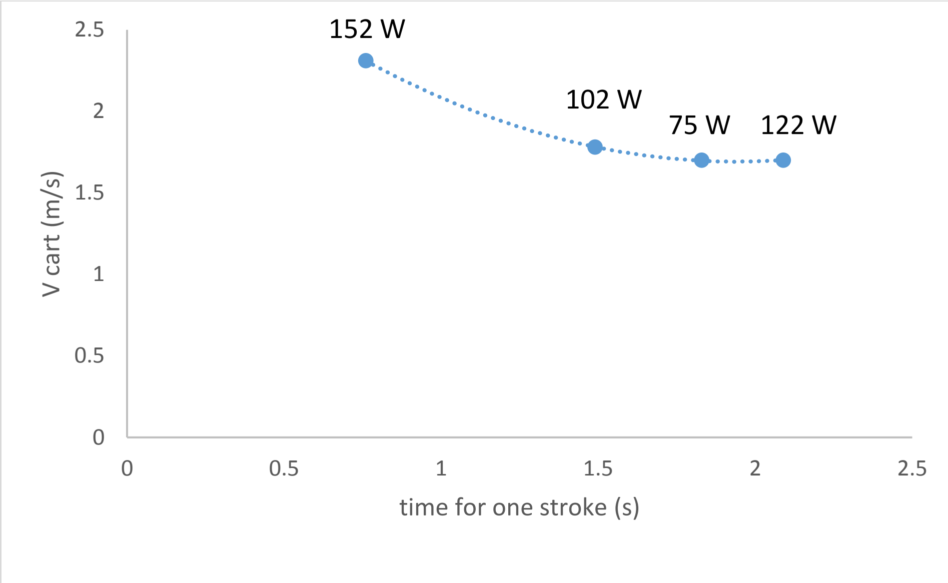

Figure 13. Wattage expended as a function of stroking speed and cart velocity.

DISCUSSION

It is at first glance remarkable that the pilots naturally stroked at St_{TE} values (0.4 and 0.5) very close to those used in nature (0.2 < St_{TE}<0.4 (Taylor et al. (2003)). Peaks in the efficiency were found in the literature and in our experiments at Strouhal numbers of about 0.3.

Humans and animals seem to naturally find this “sweet spot”. While it may be tempting to attribute this to some form of primordial connection between humans and animals, the reason that humans stroke in the most efficient range is probably that we feel very subtle feedback as we stroke. If St gets too high or too low we work harder, and the body may sense the difference.

Triantafyllou et al. (1991 and 1993) used average velocity profiles behind oscillating foils from Koochesfahani (1989) to show that the preferred St range has a theoretical basis. They demonstrated that optimal stroking efficiencies are achieved at foil oscillation frequencies that result in maximum amplification of the unstable average wake behind the foil, which occurred at 0.25< St_{TE} < 0.35. The authors also showed that many fish species stroked in exactly this range. Foils propelled by fish, birds or humans are then subject to the same physical laws which determine optimal stroking parameters. A more remarkable outcome might then have been our results falling far outside the range found in nature.

It is also instructive to look at the Strouhal number as a ratio of ½ the average stroking velocity to the speed of the craft (the frequency is the number of cycles per second, but the amplitude covers only one half of the total cycle). If we assume the stroking velocity follows a sine function, the average stroking velocity should equal about 0.64 times the maximum velocity of the stroke. We reason that the maximum speed of the stroking must be about the same as the speed of the exiting jet, as the stroking speed is driving the exiting jet. If we further assume that the propulsor is operating at peak efficiency, so that the exiting jet velocity is very close to the speed of the craft, then we may assume under ideal conditions that the maximum speed of the stroking is the same as the speed of the craft. Then,

$$St_{\max eff} = \frac{\frac{1}{2}(0.64\, Vstroke \max)}{craft\ velocity} = 0.32$$

CONCLUSIONS

Human-powered sail pumping efficiencies of about 20% were measured at Strouhal numbers (at 0.7 mast height) of 0.3 and 0.4 and St (trailing edge) of 0.4 and 0.5. This is in about the St range considered efficient in the literature as well as that used by animals in nature.

The low efficiencies of human-powered sail pumping (compared to the maximum efficiencies measured in the flapping foil literature of about 80%) can be attributed to several factors: the high maximum angles of attack used during testing, the low heave to chord ratios used, and the poor aerodynamics of a sail compared to a rigid foil.

The heave to chord ratio may be improved to some degree by using a sail with a smaller chord. The aerodynamics and maximum angle of attack can also be modified. We estimate that it may be possible to achieve human-powered efficiencies of about 70% using a rigid wing and the proper angle of attack.

Our results show that operating in the correct Strouhal number range does not necessarily preclude inefficient operation – it appears that it may guarantee only the best combination of amplitude, frequency and velocity at a given set of other parameters such as maximum angle of attack, phase angle and airfoil design.

The twist in a sail during pumping may be beneficial if the sail is properly designed to give a uniform and efficient angle of attack throughout the entire sail span. In the case of the tests performed here, the twist brought the angle of attack near the top of the sail into an efficient range. However, the angle of attack near the center of effort of the sail (where there is little or no twist) was in a range considered inefficient. It may be necessary to use a larger sail to permit lowering the angle of attack while still generating enough thrust to provide motion.

Thrust coefficients generated of 0.63 were about the same as those found in the literature at the same angle of attack.

The heart rate of the pilots was found to give power input readings that were within 15% of readings taken using an electronic force-data acquisition system. This is a very useful finding, especially when measurements need to be performed on the water or anywhere electronic equipment is impractical.

We are in the process of building a rigid sail to determine if sail pumping efficiency can be improved. Our goal is to eventually construct a small boat to test sail pumping under marine conditions and low wind speeds. It is hoped that sail pumping will be more efficient as an auxiliary power source for sailboats in light wind conditions than conventional marine propellers.

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