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# Algebraic Determination of Land HPV Velocity

John Snyder
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***Abstract***

*A method to solve for velocity when power is known.*

**Introduction**

The following equations are intended for exploring how different Human Powered Vehicle (HPV) drag parameters in conjunction with a constant power input influence the velocity result. These calculations depend on several assumptions: a constant rate of travel occurring in a forward linear direction, a zero or positive (uphill) slope, and no wind present .

**Discussion**

The total power required to propel a land HPV at a constant velocity is easily calculated when the sum of losses and gains is also known [reference1]. Solving for velocity posses a slightly greater challenge.

In the equation depicting power due to aerodynamic drag [equation 1], Cd is the aerodynamic drag coefficient, A is the vehicle's reference area, air density is the local current air density, and velocity represents the vehicle's ground speed when traveling through still air.

$$\text{Power}_{aero} = \frac{\text{Cd} \cdot \text{A} \cdot \text{air density} \cdot \text{velocity}^3}{2}$$ [equation 1]

Because velocity is presented as a number to the third power finding its value, when unknown, requires manipulating a "depressed" cubic equation of the form:

$$X = Y \cdot v^3 + Z \cdot v$$ [equation 2]

The cubic formula was "first published by Girolamo Cardano (1501-1576) in his Algebra book Ars Magna" [reference 2]. By applying several ingenious substitutions and observations Cardano made it possible to solve for velocity by employing the better-known quadratic formula as an intermediate step.

**Method**

To determine velocity begin by collecting values for the following inputs (S.I. unit): effective drag area, Cd·A ($m^2$); current local air density, air density ($kg/m^3$); combined rider and vehicle mass, mass (kg); local acceleration of gravity, g ($m/s^2$); coefficient of rolling resistance on the local surface, Crr (N/N); roadway slope, slope (radians); input power to drive train, power (watts); and drive train efficiency, efficiency (decimal).

Perform the following intermediate steps. Conclude by solving for velocity (m/s).

$$K \equiv \frac{2 \cdot mass \cdot g \cdot [sine(slope) + Crr]}{Cd \cdot A \cdot air\ density}$$ [equation 3]

$$L \equiv \frac{2 \cdot power \cdot efficiency}{Cd \cdot A \cdot air\ density}$$ [equation 4]

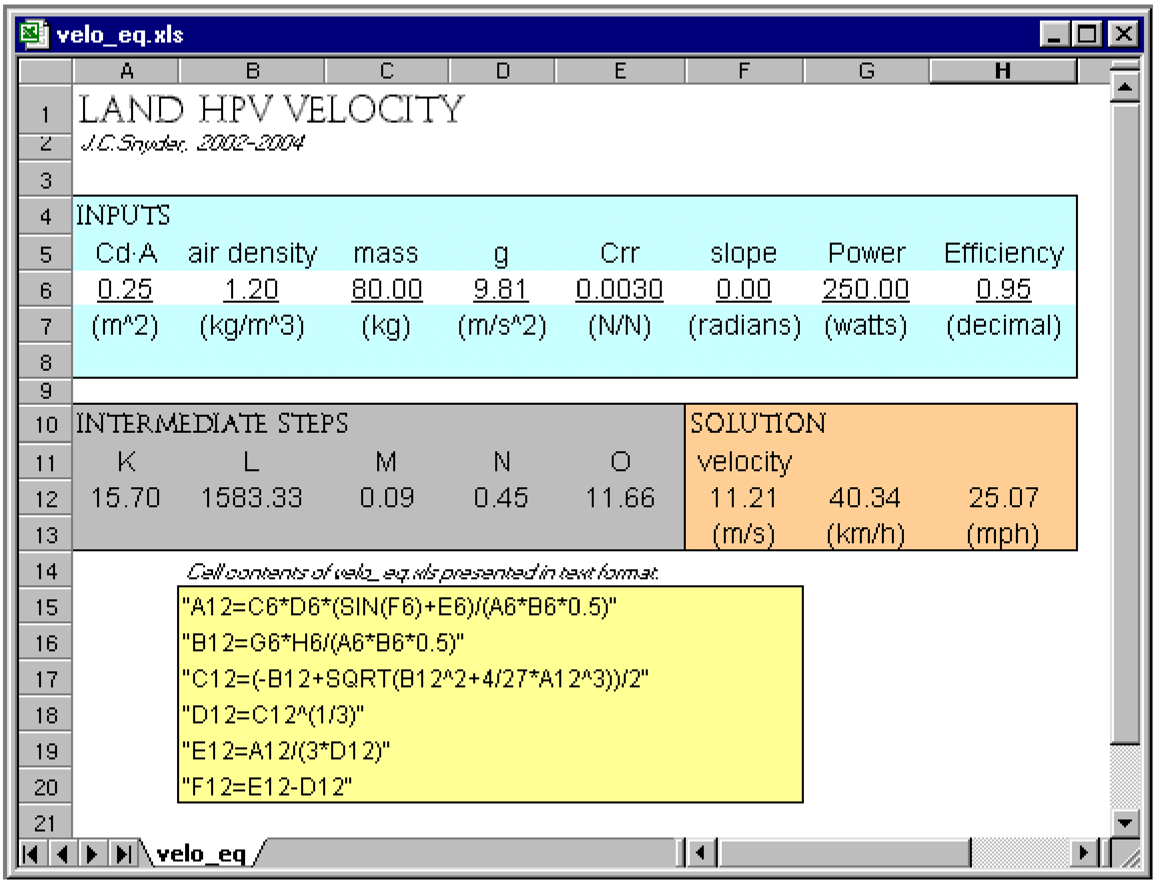
$$M \equiv \frac{-L + \sqrt{L^2 + \frac{4}{27} \cdot K^3}}{2}$$ [equation 5]

$$N \equiv M^{1/3}$$ [equation 6]

$$O \equiv \frac{K}{3 \cdot N}$$ [equation 7]

$$velocity = O - N$$ [equation 8]

**Results**


velo_eq.xls

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | LAND HPV VELOCITY | | | | | | | |
| 2 | J.C.Snyder, 2002-2004 | | | | | | | |
| 3 | | | | | | | | |
| 4 | INPUTS | | | | | | | |
| 5 | Cd·A | air density | mass | g | Crr | slope | Power | Efficiency |
| 6 | 0.25 | 1.20 | 80.00 | 9.81 | 0.0030 | 0.00 | 250.00 | 0.95 |
| 7 | (m^2) | (kg/m^3) | (kg) | (m/s^2) | (N/N) | (radians) | (watts) | (decimal) |
| 8 | | | | | | | | |
| 9 | | | | | | | | |
| 10 | INTERMEDIATE STEPS | | | | | SOLUTION | | |
| 11 | K | L | M | N | O | velocity | | |
| 12 | 15.70 | 1583.33 | 0.09 | 0.45 | 11.66 | 11.21 | 40.34 | 25.07 |
| 13 | | | | | | (m/s) | (km/h) | (mph) |
| 14 | | Cell contents of velo_eq.xls presented in text format. | | | | | | |
| 15 | | "A12=C6*D6*(SIN(F6)+E6)/(A6*B6*0.5)" | | | | | | |
| 16 | | "B12=G6*H6/(A6*B6*0.5)" | | | | | | |
| 17 | | "C12=(-B12+SQRT(B12^2+4/27*A12^3))/2" | | | | | | |
| 18 | | "D12=C12^(1/3)" | | | | | | |
| 19 | | "E12=A12/(3*D12)" | | | | | | |
| 20 | | "F12=E12-D12" | | | | | | |
| 21 | | | | | | | | |

velo_eq

[Illustration]: Screen capture of spreadsheet velo_eq.xls

## Concluding Remarks

As presented, this handling allows input only of a non-negative slope value. Notice that the determination of the value of "M" is the adaptation of the quadratic formula to a cubic equation.

## References


1. Snyder, John. 2000.CdA and Crr measurement. Human Power.51:9-13

2. Khamsi, Mohamed. Kanust, H. and Marcus, N. 2002. The "Cubic Formula". Math Medics, L.L.C. . Available from URL: http://www.sosmath.com/algebra/factor/fac11/fac11.html


## Supplemental File

Excel format spreadsheet: velo_eq.xls (20 KB)

## Disclosure Statement


The author is an associate editor of Human Power eJournal.